Formulae for Lens Selection

After selecting a line scan camera, a lens appropriate to the task must be chosen and the number and size of extension rings determined.

The most suitable lens extension and appropriate depth of focus can be calculated exactly.

Calculation is performed in the following 5 steps:

Step 1: Magnification. For a given measuring range L and a selected line scan camera with sensor length S, the magnification β can be calculated, see (F2).

Step 2: Focal length. With magnification and the given measuring distance OO' then the focal length f of a suitable lens can be calculated, see (F3).

Conversely, for a given lens focal length f and magnification β, the required measuring distance OO' can be calculated, (F6).

Step 3: Lens extension and tubes length. To achieve a sharply focussed image, the lens must be a defined distance from the line sensor.

For CCTV lenses and photo lenses, the lens extension Δs is calculated (F4) and set using the focussing mechanism itself.

For scan and macro lenses, the lens extension Δs is set using an external focussing mechanism. Additional extension rings are required for extremely large magnifications.

A required tube length LT is calculated from (F4) and (F5) and implemented with extension rings and – in the case of scan and macro lenses – with a focus adapter, as well.

Step 4: Depth of focus. For imaging objects of a certain thickness then the depth of focus $2z$ has to be determined. This must be large enough to allow fully focussed imaging of the whole measured object (F8).

Step 5: Resolution. A large F-number causes a reduction of resolution because of diffraction. The resolution of the imaging has to be controlled (F11). For magnifications $\beta > 6$, an effective F-number K' must be included instead of K (F9).

The resolution Δy_{min} must not exceed the pixel pitch of the line sensor.

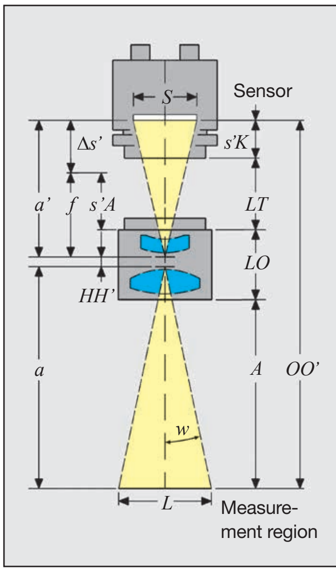


Figure 1:
Schematic depiction of the imaging system and definition of variables used

f	Lens focal length (mm)
S	Sensor length (mm)
L	Measurement region (mm)
a	Object range (mm)
a'	Image range: Distance of element to H' (mm)
β	Magnification
w	Field angle
OO'	Distance from sensor to measurement region (mm)
HH'	Principal point distance (mm) (can lengthen or shorten OO')
$s'K$	Camera flange length
$s'A$	Flange focal length (mm)
$\Delta s'$	Lens extension (mm)
LT	Tube length consisting of focus adapter FA22-... and extension rings ZR... (mm)
LO	Lens length (mm)
A	Working distance (mm)

F1: Imaging equation

Object range and image range are related by the imaging focal length:

$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}$$

The formulae F3 to F6 are derived from this equation.

F2: Magnification β:

The magnification is defined as

$$\beta = \frac{\text{sensor length}}{\text{measuring region}} = \frac{S}{L} = \frac{a'}{a}.$$

Example: Measuring region L = 290 mm, sensor length S = 28.7 mm:
$\beta = S/L = 28.7/290 = 1/10.1$

F3: Calculation of focal length f

With magnification β and distance sensor-measuring region OO'

$$f = \frac{OO'}{1/\beta + \beta + 2}$$ or for β 1/10 approximately $$f = \frac{OO'}{1/\beta + 2}.$$

Example: Magnification β = 1/10.1 and OO' = 605 mm:
Focal length f = 605 mm/ (10.1 + 2) = 50 mm

F4: Lens extension $\Delta s'$

With the magnification β and focal length f, the lens extension is $\Delta s' = f \cdot \beta$

Example 1: Magnification β = 1/10.1 and focal length f = 50 mm:
$\Delta s'$ = 50 mm / 10.1 = 4.95 mm

Example 2: In macro imaging with β = 1, (1:1 imaging)
the lens extension equals the focal length f.

F5: Tube length LT

LT = flange focal length + lens extension - (Camera flange length)

$LT = s'A + \Delta s' - s'K$

Example: Rodagon 4.0/80, focal length f = 81 mm, β = 1/6,
$s'A$ = 74.7 mm, $s'K$ = 19.5 mm:
$\Delta s' = f / \beta$ = 81 mm/ 6 = 13.5 mm
LT = 74.7 mm + 13.5 mm - 19.5 mm = 68.7 mm

Implemented by:	focus adapter FA22-40	22.0 mm
	+ focus adapter extension	6.7 mm
	+ 2x extension rings ZR20	40.0 mm
	Total =	**68.7 mm**

F6: Distance sensor - measuring region OO'

With magnification β and focal length f then

$$OO' = \left(\beta + \frac{1}{\beta} + 2\right) \cdot f + HH'$$

For $\beta \leq 1/10$ then OO' approximates $(1/\beta + 2) \cdot f + HH'$.

Example 1: Video lens B1614A, focal length f = 16 mm, HH' = 3.85 mm,
L = 290 mm, S = 13.3 mm:
$OO' = (L/S + 2) \cdot f + HH'$ = (290/13.3 + 2) ·16 mm + 3.85 mm
= 384.7 mm (as an approximation)

Example 2: Rodagon 4.0/80, focal length f = 81 mm, HH' = -2.5 mm, β = 1/6:
$OO' = (1/\beta+\beta+2) \cdot f + HH'$ = (1/6+6+2) · 81mm - 2.5 mm = 658.7 mm

F7: Field angle w

The field angle w is determined by the sensor length S, the focal length f and magnification β:

$$w = \arctan\left(\frac{S}{2 \cdot f \cdot (1 + \beta)}\right)$$

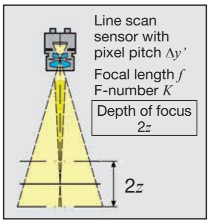
The field angle is used for calculating the edge intensity, F10.

F8: Depth of focus

The depth of focus $2z$ is calculated from

$$2z = 2 \cdot \Delta y' \cdot K \cdot \frac{1}{\beta}\left(1 + \frac{1}{\beta}\right)$$

using the F-number K, the pixel pitch (mm) $\Delta y'$, and the magnification β.



Example: Pixel pitch $\Delta y'$ = 0.014 mm
reciprocal magnific. $1/\beta$ = 10
F-number K = 4
then $2z$ = 2 · 0,014 mm · 4 · 10 · (1+10) = 12,3 mm

F-number K	2	2,8	4	5,6	8	11	22	32
Depth of Focus $2z$ [mm]*	6,2	8,6	12,3	17,2	24,6	33,9	67,8	98,6
relative signal amplitude	16	8	4	2	1	1/2	1/4	1/8

* for $\Delta y'$ = 0.014 mm and β = 1/10

F9: Effective F-number K', relative signal amplitude

For small magnifications $\beta \leq 1/10$ when calculating signal amplitude or the limit of lens resolution caused by diffraction (see F12), the F-number (focal length/ aperture diameter) is replaced by an effective F-number (image range/ aperture diameter). With a nominal F-number K and small magnification β then the effective F-number K' is calculated from:

$$K' = K \cdot (1 + \beta)$$

Example: Nominal F-number K = 4, magnification β = 1:
effective F-number $K' = 2 \cdot K = 8$

Magnification β	1/∞	1/8.2	1/5.3	1/3.8	1/2.4	1
Aperture Stops	0	+1/3	+1/2	+2/3	+1	+2
Rel. signal amplitude	1	0.79	0.71	0.63	0.5	0.25

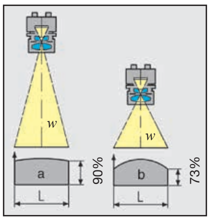
Relative signal amplitude

$$= \left(\frac{\beta}{(1 + \beta)}\right)^2$$

F10: Edge intensity

The edge intensity of line scan signals is determined by the illumination and the field angle w (see F7). Even for homogeneous illumination, the signal amplitude decreases towards the ends of the line:

Edge intensity [%] = $100 \cdot \cos^4(w)$



Rule of thumb: The focal length should equal or be more than the sensor length. In this case, the edge intensity is >70% of the center intensity.

Example: Edge intensities calculated for two different field angles using the same sensor length S = 28.7 and magnification β = 1/4:

a)	b)
focal length f = 50 mm	focal length f = 28 mm
field angle w = 13°	field angle w = 22.3°
edge intensity = 90%	**edge intensity = 73%**

F11: Diffraction limit

The resolution of a lens is limited by diffraction and declared using the effective F-number K' (see F8). The best possible resolution is achieved by closing the lens aperture by 1 to 2 steps, so that the lens resolution approaches the diffraction limit. Adjacent image elements are distinguishable, therefore, when their distance is:

$$\Delta y' \geq 2{,}4 \cdot \lambda \cdot K'$$

The optical wavelength λ for visible radiation can be considered to be 550 nm.

Example: effective F-number K' = 8
wavelength λ = 550 nm
$\Delta y'_{min}$ = 10.6 µm

effective F-number K'	Diffraction lim. Resolution* Δy_{min} [µm]
2	2,6
2,8	3,7
4	5,3
5,6	7,4
8	10,6
11	14,5
16	21,1
22	29,0

*at wavelength λ = 550 nm

Schäfter+Kirchhoff GmbH
Kieler Str. 212, 22525 Hamburg, Germany • Tel: +49 40 85 39 97-0 • Fax: +49 40 85 39 97-79 • info@SuKHamburg.de • www.SuKHamburg.com